METHOD OF DETERMINING OPTIMAL SPACECRAFT TRAJECTORY CORRECTION

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16. Abstract

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METHOD OF DETERMINING OPTIMAL SPACECRAFT TRAJECTORY CORRECTION

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ABSTRACT. We examine an optimal spacecraft trajectory correction method consisting of determining the optimal energetics and strategy for interplanetary spacecraft trajectory correction. Calculation results for the Earth-Jupiter flight trajectory are presented. The feature which distinguishes this method from those developed previously involves account for orbit determination errors based on trajectory measurements and the correcting impulse execution errors.

Introduction

Spacecraft [SC] which are launched at the present time are /3*
equipped with motion parameter measurement systems, orientation and stabilization systems, and power plants which permit control of the motion in outer space. One of the control modes is trajectory correction — impulsive variation of the SC velocity accomplished with the objective of correcting the deviations of the basic trajectory parameters from their values corresponding to the nominal design flight trajectory.

*Numbers in the margin indicate pagination in the original foreign text.

In general form, the correction problem can be formulated as follows. As a result of SC motion observation, we establish on the basis of trajectory measurements the discrepancies between the parameters of the actual and nominal trajectories. If the discrepancies exceed the allowable values, then for specified or variable engine firing times we must determine the direction and magnitude of the SC velocity change, as a result of which the indicated discrepancies are reduced and will not exceed the allowable values.

The energetic characteristic of the correction is the total impulse. By correction strategy, we mean the rule specifying the correcting impulses on the basis of trajectory measurements and SC trajectory prediction.

2. Basic Parameters and Relations

On the basis of [1], we introduce the correcting and corrected vectors and determine the generalized linear correction, with account for correction execution errors.

We denote by I_N the set of indices $\{1, 2 \ldots N\}$. With each element k of the set I_N, we associate the Euclidean space K_k of dimension n_k /4
— the space of the correcting vectors at the point k, whose elements are the vector $\bar{V}_k = \{V_{k1} \ldots V_{kn_k}\}^T$ with the norm:

$$w_k = \|\bar{V}_k\| = \sqrt{\sum_{s=1}^{n_k} V_{ks}^2} \tag{1}$$

and the vector $\delta\bar{V}_k = \{\delta V_{k1} \ldots \delta V_{kn_k}\}$ characterizing the $\bar{V}_k$ "execution" error. For each k, we examine the Euclidean space $R^m(\xi_1 \ldots \xi_m)$ of constant dimension m, in which there appear the corrected parameter vectors:

$$\bar{\xi}_k = \{\xi_k^{(1)} \ldots \xi_k^{(m)}\}^T, \quad k = 1 \ldots N$$

If the matrices of dimension A_k are given (the methods for calculating the A_k matrices were presented in [1, 2]), the N linear mappings

$R_k^{n_k} \rightarrow R^m$:

$$\bar{\xi}_K = A_K(\bar{V}_K + \delta\bar{V}_K) \qquad (2)$$

determine the generalized and linear orbital correction. To every k, we place in correspondence the state vector $\bar{x}_K = \{x_K^{(1)} \ldots x_K^{(m)}\}$ $\bar{x}_K \in R^m$. The components of the vector $\bar{x}_k$ may be, for example, the planetocentric coordinates of a SC performing an interplanetary mission, and the deviation from the normal value of the time of SC approach to the target planet after application of K correction impulses. In addition, we introduce the initial state vector $\bar{x}_0 = \{x_0^{(1)}, \ldots, x_0^{(m)}\}^T$. We denote the nominal value of $\bar{x}_0$ by b. We assume that the upper bounds $A_{0i}(i = 1 \ldots m)$ of the moduli of the components of the vector $\bar{x}_0 - \bar{b}$ are known a priori, the inequalities $|x_0^{(i)} - b_i| \leq A_{0i}$ in this case define the region $\Omega_0 \in R^m$. We examine as the corrected parameter vector:

$$\bar{\xi}_K = \bar{x}_K - \bar{x}_{K-1}. \qquad (3)$$

From (2) and (3), we obtain the equalities:

$$\bar{x}_K = \bar{x}_{K-1} + A_K(\bar{V}_K + \delta\bar{V}_K) \qquad (4)$$

We denote by $\bar{y}_{K-1} \in R^m$ the estimate of the vector $\bar{x}_{K-1}$, determined on the basis of trajectory measurements on the time interval $(0, K)$. Then the relation is valid /5

$$\bar{y}_{K-1} = \bar{x}_{K-1} + \delta\bar{x}_{K-1}, \qquad (5)$$

in which $\delta\bar{x}_{K-1}$ is the error of determination of the vector $\bar{x}_{K-1}$,

$$\delta\bar{x}_{K-1} \in R^m, \quad \delta\bar{x}_{K-1} = \{\delta x_{K-1}^{(1)} \ldots \delta x_{K-1}^{(m)}\}^T$$

Application of optimal orbit determination strategy [3] permits finding $\ell_{K-1}^{(i)} = \min_{\beta} \max_{\rho_j} |\delta x_{K-1}^{(i)}|$, where β is the ensemble of all possible measurement compositions for the given strategy, and ρ_j are bounded errors of measurement of the function $\mathcal{D}_j(\bar{q}_0, \bar{V}_1 \ldots \bar{V}_{K-1})$, which depends on the parameters $\bar{q}_0, \bar{V}_1 \ldots \bar{V}_{k-1}$ which determine uniquely the trajectory and the a priori information on these parameters. Let us examine the set $R_{k-1} \in R^m$ of vectors $\delta\bar{x}_{k-1}$ for the components of which the inequality is satisfied:

$$|\delta x_{k-1}^{(i)}| \leq \ell_{k-1}^{(i)} \tag{6}$$

We further let $\bar{z}_k = \{z_k^{(1)} \ldots z_k^{(m)}\}^T$ be the "sighting" vector for the k^{th} correction impulse, connected with the vectors $\bar{y}_{k-1}, \bar{V}_k$ and the matrix A_k by the relation:

$$\bar{z}_k = \bar{y}_{k-1} + A_k \bar{V}_k \tag{7}$$

From (5) and (7) follows the equality:

$$\bar{z}_k = \bar{x}_{k-1} + \delta\bar{x}_{k-1} + A_k \bar{V}_k \tag{8}$$

We assume that the components of the vectors $\delta\bar{V}_k$ are represented as:

$$\delta V_{ks} = \bar{\ell}_{ks} \cdot \bar{V}_k, \quad s = 1 \ldots n_k, \tag{9}$$

where $\bar{\ell}_{ks}$ is a random vector of dimension n_k. From the various components of the vector $\{\bar{\ell}_{k1} \ldots \bar{\ell}_{kn_k}\}$, we form the vector $\bar{\mathcal{L}}_k = \{\mathcal{L}_{k1} \ldots \mathcal{L}_{k\nu}\}\ (\nu \leq n_k^2)$, and consider that its components are bounded:

$$|\mathcal{L}_{k\ell}| \leq \lambda_{k\ell}, \quad \ell = 1 \ldots \nu. \tag{10}$$

The inequalities (10) define the region Λ_k in V — the Euclidian space measure. /6

By $\bar{V}_k$, we mean either [4] the minimal with respect to norm (I) (with the condition $\operatorname{rang} A_k = m$), or the solution of (8) determined using the Cramer rule (with $m \leq n_k$ и $m > n_k$):

$$\bar{V}_k = A_k^T (A_k A_k^T)^{-1} (\bar{z}_{k-1} - \bar{y}_{k-1}), \quad \bar{V}_k = \mathcal{A}_k (\mathcal{B}_k \bar{z}_k - \bar{y}_{k-1}), \tag{11}$$

in which A_k is a known matrix of dimension $n_k \cdot m$, β_k is a matrix of dimension $m \cdot m$, expressing the n_k free components $\bar{z}_k$:

$$\beta_k = \begin{pmatrix} E_{n_k} & O\,(n_k \times (m-n_k)) \\ O\,((m-n_k) \times m) & \end{pmatrix}$$

where $E_{n_k}, O(n_k \times (m-n_k)), O((m-n_k)\times m)$, respectively, are a $n_k \times n_k$ unit matrix and $n_k \times (m-n_k)$, $(m-n_k) \times m$ null matrices. We shall assume that trajectory measurements processing is accomplished by the maximal likelihood method. The computational formulas for the estimate $\bar{y}_{k-1}$ and covariation matrix $K(\delta\bar{x}_{k-1})$ are presented in [5] and [1]. As is known, the mean-square errors of the components of the vector $\delta\bar{x}_{k-1}$ are:

$$\sigma(\delta x_{k-1}^{(i)}) = \sqrt{(K(\delta\bar{x}_{k-1}))_{ii}}$$

We introduce the following sets of vectors $\delta\bar{x}_{k-1}$:

$$\mathcal{D}_{k-1}: \delta x_{k-1} \,/\, |\delta x_{k-1}^{(i)}| \leq \sigma(\delta x_{k-1}^{(i)})$$

3. Optimality Criterion

It follows from (4), (9), and (11) that $\bar{x}_k$ and $\bar{V}_k$ depend on the parameters $\bar{x}_0, \delta\bar{x}_0 \ldots \delta\bar{x}_{k-1}, \bar{z}_1 \ldots \bar{z}_k, \bar{\mathcal{L}}_1 \ldots \mathcal{L}_{k-1}$, while x_k also depends on $\bar{\mathcal{L}}_k$. In R^m we specify the region Ω $(\Omega \subset \Omega_0)$ with the aid of the inequalities $|\xi_i - \ell_i| \leq B_i$, B_i — the maximal allowable discrepancies. Under the assumptions made, we determine for $\bar{x}_0 \in \Omega_0$, $\delta\bar{x}_{k-1} \in R_{k-1}$, $\bar{\mathcal{L}}_k \in \Lambda_k$ the best guaranteed total correction impulse estimate:

$$I = \max_{\bar{x}_0} \max_{\delta\bar{x}_0} \min_{\bar{z}_1} \max_{\bar{\mathcal{L}}_1} \max_{\delta\bar{x}_1} \ldots \max_{\bar{\mathcal{L}}_{N-1}} \max_{\delta\bar{x}_{N-1}} \min_{\bar{z}_N} \left(\sum_{k=1}^{N} w_k\right) \tag{12}$$

with the conditions on the choice of $\bar{z}_1 \ldots \bar{z}_N$ for each given $\bar{x}_0$: /7

$$g_i(\bar{x}_0, \bar{z}_1 \ldots \bar{z}_N) = \max_{\delta\bar{x}_0, \bar{\mathcal{L}}_1 \ldots \delta\bar{x}_{N-1}, \bar{\mathcal{L}}_N} |x_N^{(i)} - \ell_i| \leq B_i, \quad i = 1 \ldots m. \tag{13}$$

Estimate (9) corresponds to successive SC trajectory correction with account for the errors characterizing:

(1) SC injection into orbit;

(2) determination of SC orbit on the basis of trajectory measurements;

(3) execution of the correcting impulses.

Control effectiveness is characterized by selection of the parameters $\bar z_n \; (n = 1 \dots N)$. Let us examine the following $\bar z_n$ selection procedure, specifying the correction strategy. For the $\bar y_0$ and $\sigma(\delta x_0^{(i)}) \; (i = 1 \dots m)$ determined as a result of trajectory measurement processing, we first calculate the $\bar z_1$ for which:

$$I_1(\bar y_0) = \max_{\delta \bar x_0} \min_{\bar z_1} \dots \max_{\delta \bar x_{N-1}} \min_{\bar z_N} \left(\sum_{k=1}^{N} w_k \right)$$

for $\delta \bar x_0 \in D_0$ and $\bar z_1 \dots \bar z_N$, satisfying the inequalities

$$g_{1i}(\bar y_0, \bar z_1 \dots \bar z_N) = \max_{\delta \bar x_0 \dots \bar \alpha_N} \left| M_{\bar x_0}(x_N^{(i)}) - b_i \right| \le B_i,$$

where M is the mathematical expectation symbol. Then in order to determine $\bar y_0 \dots \bar y_{n-1}$ and $\sigma(\delta x_{n-1}^{(i)})$, and also in order to calculate $\bar z_1 \dots \bar z_{n-1} \; (w_1 \dots w_{n-1})$, we find the $\bar z_n$ for which

$$I_n(\bar y_0 \dots \bar y_{n-1}, \bar z_1 \dots \bar z_n) = \max_{\delta \bar x_0} \max_{\bar \alpha_1} \dots \min_{\bar z_n} \min_{\bar z_N} \sum_{k=1}^{N} w_k$$

for $\bar z_n \dots \bar z_N$, satisfying the inequalities:

$$g_{ni}(\bar y_0, \bar z_1 \dots \bar z_N) = \max_{\delta \bar x_0 \dots \bar \alpha_N} \left| M_{\bar x_0}(x_N^{(i)}) - b_i \right| \le B_i$$

in (I_n) and (g_{ni}) $\delta \bar x_{\ell-1} \in D_{\ell-1}$, $\ell \le n$, $\delta \bar x_k \in R_k$, $\bar \alpha_\ell \in \Lambda_\ell$, $\alpha_k \in \Lambda_k$.

The $\bar z_n$ selection problem is mathematically analogous to the Problem (12) - (13).

We note that calculation of I_n reduces to minimization of w_N /8
with respect to $\bar{z}_N$.

4. Optimal Parameter Theorem

Inequalities (13) define the set $Q_{\bar{x}_0}(\bar{z}_1 \ldots \bar{z}_N)$ of dimension N · m, for which the lemma holds: the set $Q_{\bar{x}_0}(\bar{z}_1 \ldots \bar{z}_N)$ is convex. For definiteness, in view of the convexity of $Q_{\bar{x}_0}$, we shall consider that in $Q_{\bar{x}_0}$ there is inscribed the "beam" $B_{\bar{x}_0}(\bar{z}_1 \ldots \bar{z}_N)$ $(B_{\bar{x}_0} \subseteq Q_{\bar{x}_0})$: the set of vectors $\{\bar{z}_1 \ldots \bar{z}_N\}$, for the components of which the inequalities are satisfied

$$\alpha_{ki} \leq z_{ki} \leq \beta_{ki}, \quad k = 1 \ldots N, \; i = 1 \ldots m$$

In this case, $B_{\bar{x}_0} = B_{1\bar{x}_0} + \ldots + B_{N\bar{x}_0}$,
($B_{\bar{x}_0}$ is the direct sum of the sets $B_{k x_0} \in Q_{\bar{x}_0})$, $B_{k\bar{x}_0}(\bar{z}_k)$: $\{\bar{z}_1 \ldots \bar{z}_N\} \alpha_{ki} \leq z_{ki} \leq \beta_{ki}$, $\bar{z}_{\bar{k}} = \bar{\beta}$ for $\bar{k} \neq k$. We denote the boundary of the set $B_{k\bar{x}_0}$ by $\tilde{B}_{k\bar{x}_0}$.

In finding I, we perform minimization with respect to $\bar{z}_k$ on the sets $B_{k x_0}$.

We denote by Ω_0^*, R_{k-1}^* and Λ_k^*, the sets of boundary points of the regions Ω_0, R_{k-1} and Λ_k.

On the basis of [6] - [9], the mathematical solution of the problems posed in Section 3 leads to the following theorems.

Theorem 1: the extremal value of I is reached for

$$\bar{x}_0 \in \Omega_0^*, \; \delta\bar{x}_{k-1} \in R_{k-1}^*, \; \bar{y}_k \in \Lambda_k^*$$

Theorem 2: the extremal value is reached for

$$\bar{z}_k \in \tilde{B}_{k x_0}$$

Theorem 3: for $w_1 \ldots w_{n-1}, I_n \ (n=1, N)$ with the conditions $\delta x_{n-1} \in Q_{n-1}$, the inequality $w_1 + \ldots + w_{n-1} + I_n \leq I$ is satisfied.

5. Algorithm for Calculating the Optimal Correction Energetics Estimate

Let us examine two-impulse (N = 2) two-parameter (m = 2) interplanetary SC trajectory corrections in which the engine can be oriented arbitrarily ($n_k = 3$). We shall consider that the transfer time T is fixed. /9

In accordance with [2], we write (11) in the form

$$\bar{V}_k = \bar{G}_k \left(x_{k-1}^{(1)} + \delta x_{k-1}^{(1)} - z_k^{(1)} \right) + \bar{F}_k \left(x_{k-1}^{(2)} + \delta x_{k-1}^{(2)} - z_k^{(2)} \right), \tag{14}$$

where $\bar{G}_k$ and $\bar{F}_k$ are equal to

$$\bar{G}_k = \frac{\bar{A}_k^2 \times \bar{A}_k^1 \times \bar{A}_k^2}{|\bar{A}_k^1 \times \bar{A}_k^2|^2}, \quad \bar{F}_k = \frac{\bar{A}_k^1 \times \bar{A}_k^2 \times \bar{A}_k^1}{|\bar{A}_k^1 \times \bar{A}_k^2|^2},$$

$\bar{A}_k^1, \bar{A}_k^2$ are, respectively, the first and second rows of the matrix A_k.

It follows from vector algebra that the equalities are satisfied

$$\bar{A}_k^1 \cdot \bar{G}_k = 1, \quad \bar{A}_k^2 \cdot \bar{F}_k = 1, \quad \bar{A}_k^2 \cdot \bar{G}_k = 0, \quad \bar{A}_k^1 \cdot \bar{F}_k = 0 \tag{15}$$

We take the following correcting impulse execution error model [10]:

$$\delta \bar{V}_k = \bar{\varepsilon}_k \times \bar{V}_k + \beta_k \bar{V}_k, \tag{16}$$

where $\bar{\varepsilon}_k$ is the vector of correcting velocity direction angular error, β_k is the proportional error in the correcting increment magnitude, and $|\varepsilon_k^{(\mu)}| \leq e_{k\mu}$, $\mu = 1,2,3$, $|\beta_k| \leq \sigma_k$. With the notations used, $\xi_k = \{\varepsilon_k^{(1)}, \varepsilon_k^{(2)}, \varepsilon_k^{(3)}, \beta_k\}$. We note that each set Ω_0^*, R_{k-1}^* $(k = 1,2)$ consists in the present case of four vectors and Λ_k^* consists of 16. From (15), (14), (8), (4), and (16), we obtain:

$$x_k^{(1)} = z_k^{(1)} - \delta x_{k-1}^{(1)} + \beta_k (z_k^{(1)} - x_{k-1}^{(1)} - \delta x_{k-1}^{(1)}) + (z_k^{(2)} - x_{k-1}^{(2)} - \delta x_{k-1}^{(2)}) \bar{\varepsilon}_k \cdot (\bar{\Lambda}_k \times \bar{F}_k), \tag{17}$$

$$x_k^{(2)} = z_k^{(2)} - \delta x_{k-1}^{(2)} + \beta_k (z_k^{(2)} - \delta x_{k-1}^{(2)} - x_{k-1}^{(2)}) + (z_k^{(1)} - x_{k-1}^{(1)} - \delta x_{k-1}^{(1)}) \bar{\varepsilon}_k \cdot (\bar{A}_k^2 \times \bar{G}_k) \tag{18}$$

From (17) and (18), we obtain: /10

$$x_2^{(1)} = f_1 (\bar{x}_0, \delta\bar{x}_0, \delta\bar{x}_1, \bar{\mathcal{X}}_1, \bar{\mathcal{X}}_2, \bar{z}_1, \bar{z}_2), \tag{19}$$

$$x_2^{(2)} = f_2 (\bar{x}_0, \delta\bar{x}_0, \delta\bar{x}_1, \bar{\mathcal{X}}_1, \bar{\mathcal{X}}_2, \bar{z}_1, \bar{z}_2) \tag{20}$$

On the basis of Theorems [1, 2] and the Relations (1), (14), (19), (20), we examine the following computational algorithm.

For each $\bar{x}_0 \in \Omega_0^*$ on the basis of (19) and (20), we determine the size of the set $Q\bar{x}_0$ for each measurement and plot the "beam" $B\bar{x}_0 \subset Q\bar{x}_0$. We then apply the sequential maximum calculation method with use of interpolation with respect to $\bar{z}_1$ on the set $\tilde{B}_1\bar{x}_0$ (we denote the set of interpolation nodes $B_1^*\bar{x}_0$). The calculations are made for $x_0 \in \Omega_0^*$, $\delta\bar{x}_{k-1} \in R_{k-1}^*, \bar{\mathcal{X}}_k \in \Lambda_k^*, \bar{z}_k \in \tilde{B}_k$, using the following scheme.

$$\min_{\tilde{B}_{2\bar{x}_0}} w_2 \rightarrow \max_{\Lambda_1^* R_1^*} \min_{\tilde{B}_{2\bar{x}_0}} w_2 \rightarrow \min_{B_{1\bar{x}_0}^*} \max_{\Lambda_1^* R_1^*} \min_{\tilde{B}_{2\bar{x}_0}} (w_1 + w_2) \rightarrow$$

$$\xrightarrow[\text{w. respect to } \bar{z}_1]{\text{interpolation}} \max_{\Omega_0^*, R_0^*} \min_{\tilde{B}_{1\bar{x}_0}} \max_{\Lambda_1^*, R_1^*} \min_{\tilde{B}_{2\bar{x}_0}} (w_1 + w_2) = J.$$

6. Numerical Examples

The quantity I was determined for given nominal SC trajectory from the Earth to Jupiter with launch data August 15, 1976 and T = 448 days. As the corrected vector, we selected (ξ, η) in the target planet picture plane [2].

The initial numerical data are presented in Tables 1 and 2: /11
t_1 and t_2 denote the times of application of the first and second correction impulses in days, the elements of the matrix A_k, denoted by A (11) ... A (23), were calculated along the nominal trajectory

TABLE 1.*

t_1, t_2	$A[11]$	$A[12]$	$A[13]$	$A[21]$	$A[22]$	$A[23]$
60	-I,4I5	2,0I7	0,908	0,044	-0,952	2,I92
I80	-0,92	I,809	0,805	0,047	-0,859	I,983
300	-0,565	I,036	0,46I	0,028	-0,50I	I,II6

*Commas represent decimal points.

TABLE 2.*

(t_1, t_2)	$\ell_0^{(1)}$	$\ell_0^{(2)}$	$\ell_1^{(1)}$	$\ell_1^{(2)}$
(60,I80)	996	I4I8	62	I4I5
(60,300)	996	I4I8	33	I393
(180,300)	42	1358	29	1338

*Commas represent decimal points

at 10^7 sec, the optimal prognoses $\ell_0^{(1)}, \ell_0^{(2)}, \ell_1^{(1)}, \ell_1^{(2)}$ were calculated for the corresponding times in hundreds of kilometers. The following were taken as the initial numerical values:

$$A_{01} = II7I \times I0^3 \text{ км}, \quad A_{02} = I023 \times I0^3 \text{ км}; \quad B_1 = B_2 = I50 \times I0^3 \text{ км};$$
$$e_{1\mu} = e_{2\mu} = 0{,}00I; \; \sigma_1 = \sigma_2 = 0{,}005; \quad b_1 = b_2 = 0 \text{ км}.$$

The results of the numerical solution are summarized in Table 3, where I, w_1, w_2 are in meters per second, the components of the vectors $\bar{x}_0, \bar{z}_1, \bar{z}_2$ and $g_1(\bar{x}_0, \bar{z}_1, \bar{z}_2), g_2(\bar{x}_0, \bar{z}_1, \bar{z}_2)$ are in hundreds of kilometers.

TABLE 3.*

(t_1, t_2)	J, w_1, w_2	$\bar{x}_0$	$\bar{z}_1$	$\bar{z}_2$	g_1, g_2
(60,I80)	60,38,22	II7I0,-I0230	4848,-3476	355,-I7	455,I472
(60,300)	80,4I,39	-II7I0,I0230	-3I82,4578	362,23	429,I460
(I80,300)	I00,30,70	-II7I0,-I0230	-5I08,-74I2	365,-37	437,I434

*Commas represent decimal points.

We see from Table 3 that:

1) early trajectory correction with t_1 = 60 days and t_2 = 180 days is energetically advantageous;

2) with increase of t_2, the ratio w_1/I decreases.

We note the suggestion in [11] of the necessity for a characteristic velocity margin on the order of 100 - 300 m/sec for trajectory correction in order to hit Jupiter. The time required to calculate the optimal parameters for a single pair (t_1, t_2) (Table 3) on the BESM-4 electronic computer was about 55 minutes. /12

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